## Collective path connecting the oblate and prolate local minima in proton-rich N = Z nuclei around $^{68}\text{Se}$

M. Kobayasi<sup>1,a</sup>, T. Nakatsukasa<sup>2</sup>, M. Matsuo<sup>3</sup>, and K. Matsuyanagi<sup>1</sup>

<sup>1</sup> Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan

<sup>2</sup> Institute of Physics, University of Tsukuba, Tsukuba 305-8571, Japan

<sup>3</sup> Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

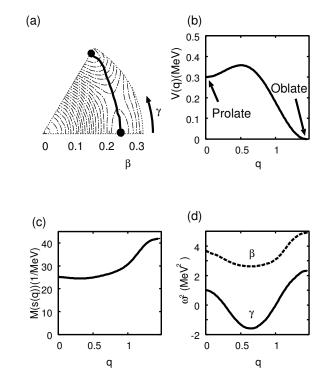
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**Abstract.** By means of the adiabatic self-consistent collective coordinate method and the pairing-plusquadrupole interaction, we have for the first time obtained a self-consistent collective path connecting the oblate and prolate local minima in  $^{68}$ Se and  $^{72}$ Kr. This self-consistent collective path is found to run approximately along the valley connecting the oblate and prolate local minima in the collective potential energy landscape. The result of this calculation clearly indicates the importance of triaxial deformation dynamics in oblate-prolate shape coexistence phenomena.

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Shape coexistence phenomena are typical examples of large-amplitude collective motion in nuclei. These phenomena implies that different solutions of the Hartree-Fock-Bogoliubov (HFB) equations (local minima in the deformation energy surface) appear in the same energy region and that the nucleus exhibits large-amplitude collective motion connecting these different equilibrium points. Recently, we have proposed a new method of describing such large-amplitude collective motion, which is called adiabatic self-consistent collective coordinate (ASCC) method [1,2]. This method is formulated on the basis of the time-dependent HFB theory (TDHB), and consists of two basic equations: 1) the HFB equation in the moving frame and 2) the local harmonic equations in the moving frame, abbreviated to the "moving frame QRPA" below. It does not assume a single local minimum, so that it is expected to be suitable for the description of the shape coexistence phenomena. The ASCC method also enables us to self-consistently include the pairing correlations, removing the spurious number fluctuation modes.

Quite recently, with use of the pairing-plus-quadrupole (P+Q) interaction, we have applied the ASCC method to the shape coexistence phenomena in <sup>68</sup>Se and <sup>72</sup>Kr, discovered by Fischer *et al.* [3] and Bouchez *et al.* [4], where the oblate ground band and the prolate excited bands compete in energy, and investigated the collective path connecting the oblate and prolate local minima in the collective potential energy landscape.



**Fig. 1.** The collective path (a), the collective potential (b), the collective mass (c), and the local eigen-frequencies squared,  $\omega^2(q)$  (d), of the moving frame QRPA as functions of the collective coordinate q, obtained by the ASCC calculation for <sup>68</sup>Se.

<sup>&</sup>lt;sup>a</sup> e-mail: kobayasi@ruby.scphys.kyoto-u.ac.jp

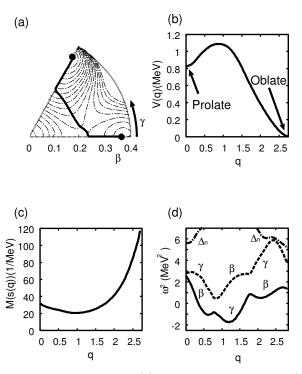


Fig. 2. The collective path (a), the collective potential (b), the collective mass (c), and the local eigen-frequencies squared,  $\omega^2(q)$  (d), of the moving frame QRPA as functions of the collective coordinate q, obtained by the ASCC calculation for <sup>72</sup>Kr.

The result of the calculation is shown in figs. 1 and 2. For <sup>68</sup>Se, because the  $\gamma$ -vibrational mode is the lowest frequency and most collective QRPA mode at the prolate and oblate local minima, we have chosen this mode as the initial condition, and successfully obtained the collective path connecting the oblate and prolate local minima. which is plotted in fig. 1(a). As we have extracted the collective path in the TDHB phase space, which has a very large number of degrees of freedom, the path drawn in this figure should be regarded as a projection of the collective path onto the  $(\beta, \gamma)$ -plane. Roughly speaking, the collective path goes through the valley that exists in the  $\gamma$  direction and connects the oblate and prolate minima. The potential energy curve V(q) along the collective path is displayed in fig. 1(b). Because the collective Hamiltonian,  $\mathcal{H}_{coll} = \frac{1}{2}B(q)p^2 + V(q)$ , is invariant under a point transformation,  $q \to q' = q'(q)$ ,  $p \to p' = p(\partial q'/\partial q)^{-1}$ , with  $B(q) \to B(q')(\partial q'/\partial q)^{-2}$ , we can take the scale of q such that the collective mass is given by  $M(q) = B(q)^{-1} =$  $1 \,\mathrm{MeV^{-1}}$ . The collective mass as a function of the geometrical length s along the collective path in the  $(\beta, \gamma)$ plane is then given by  $M(s(q)) = M(q)(ds/dq)^{-2}$ , with  $ds^2 = d\beta^2 + \beta^2 d\gamma^2$ . This quantity is plotted in fig. 1(c) as a function of q. We see an appreciable increase of the collective mass in the vicinity of  $\gamma = 60^{\circ}$ . This property might contribute to increase the stability of the oblate shape in the ground state. The eigen-frequencies of the moving frame QRPA along the collective path are plotted in fig. 1(d). The solid curve represents the frequency squared,  $\omega^2(q)$ , given by the moving frame QRPA, which corresponds to the  $\gamma$ -vibration in the oblate and prolate limits. This solution determines the infinitesimal generators  $\hat{Q}(q)$  and  $\hat{P}(q)$  along the collective path. For reference, we also present in this figure another solution of the moving frame QRPA, which possesses the  $\beta$ -vibrational properties and is irrelevant to the collective path in the case of <sup>68</sup>Se.

In contrast to  $^{68}$ Se, the lowest-frequency QRPA mode is the  $\beta$ -vibration at the prolate local minimum in <sup>72</sup>Kr. The collective path first goes in the direction of the  $\beta$ -axis in the  $(\beta, \gamma)$ -plane (see fig. 2(a)). As we go along the  $\beta$ -axis, we encounter a situation in which the two solutions (labeled by  $\beta$  and  $\gamma$ ) of the moving frame QRPA compete in energy, and they eventually cross at  $q \sim 0.8$ (see fig. 2(d)). Using an efficient algorithm developed in ref. [5] to determine the collective path in the crossing region, we have successfully obtained the smooth deviation of the direction of the collective path from the  $\beta$ -axis toward the  $\gamma$  direction. We see that the properties of the lowest mode gradually changes from those of the  $\beta$  to the  $\gamma$ -vibrations. After a smooth turn in the  $\gamma$  direction, the collective path approaches the  $\gamma = 60^{\circ}$  axis. Then, we again encounter the crossing at  $q \sim 1.8$ , where the properties of the lowest solution of the moving frame QRPA change smoothly from those of the  $\gamma$ -vibrational to those of the  $\beta$ -vibrational case. The collective path thus runs along the  $\gamma = 60^{\circ}$  axis and it finally reaches the oblate minimum. We have also carried out a calculation starting from the oblate minimum and proceeded in the inverse manner, obtaining the same collective path. The potential energy curve V(q) and mass parameter M(s(q)) along the collective path is displayed in fig. 2(b) and (c), respectively. We notice again a significant increase of M(s(q)) in the vicinity of the oblate minimum. Quite recently, Almehed and Walet found a collective path going from the oblate minimum over a spherical energy maximum into the prolate secondary minimum [6]. We also obtained such a collective path when we imposed axial symmetry on the solutions of the moving frame HB equation and always used only K = 0 solutions of the moving frame QRPA.

For the first time, the self-consistent collective paths between the oblate and prolate minima have been obtained in realistic situations starting from the microscopic P + Q Hamiltonian. The result of our calculation strongly indicates the necessity of taking into account the  $\gamma$  degree of freedom, for the description of the oblate-prolate shape coexistence phenomena in  $^{68}$ Se and  $^{72}$ Kr.

## References

- M. Matsuo, T. Nakatsukasa, K. Matsuyanagi, Prog. Theor. Phys. 103, 959 (2000).
- M. Kobayasi, T. Nakatsukasa, M. Matsuo, K. Matsuyanagi, Prog. Theor. Phys. **110**, 61 (2003).
- S.M. Fischer *et al.*, Phys. Rev. Lett. **84**, 4064 (2000); Phys. Rev. C **67**, 064318 (2003).
- 4. E. Bouchez et al., Phys. Rev. Lett. 90, 082502 (2003).
- M. Kobayasi, T. Nakatsukasa, M. Matsuo, K. Matsuyanagi, Prog. Theor. Phys. 113, 129 (2005), nucl-th/0412062.
- 6. D. Almehed, N.R. Walet, Phys. Lett. B 604, 163 (2004).